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HYDRODYNAMICS OF MIXING OF LIQUID–SOLID SYSTEMS

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We present a model of the process of distribution of solid particles over the volume of a mixer. We analyze the kinetics of suspension, and show there is agreement between experimental and calculated data.

In carrying out reactions it is necessary to know the solution of the problem of mixing of solid particles in a liquid. A major part of the investigation of this problem seemed to be the determination of the operating conditions of a mixer to ensure suspension of the solid particles. These conditions were described by the minimum rotational speed n_0 of the mixer [1-4], of the specific power $\varepsilon_V = N/V_a$ [5, 6]. The effect of the size of the devices was studied in detail in [1-3] and in [7]. However, the kinetics of the formation of a suspension, and the effect of mixing on the homogeneity of the particle distribution in the active volume of a reactor were hardly studied, although these phenomena are important in carrying out chemical transformations such as condensation polymerization.

In the present article we study the hydrodynamics of mixing of liquid–solid systems on the basis of the circulation cell model proposed earlier [8]. According to this model the active volume of the reactor is divided into cells by intersecting horizontal and vertical planes. The displacement of a liquid particle and its macromixing are described by a system of material balance equations formulated for each cell, taking account of flow conditions. The flow rates through the cell faces are determined from the calculated velocities w_φ, w_r, w_z of the medium at the plane faces of the cells by using an analytic model of the spatial hydrodynamics of mixers [9].

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Studies of single-phase systems showed that the flow of an agitated medium has a complex circulational character [10, 11], so solid particles in a liquid must perform three-dimensional circulatory motion. This can be taken into account in the model of macromixing processes [8] by introducing into the material balance equations of the cells additional flow rates of the solid phase through the cell faces produced by the motion of particles having a velocity u relative to the liquid:

$$Q_i^{(het)} = \int\int_{\sigma} u d\sigma. \tag{1}$$

We consider the relative motion of solid particles of radius 10^{-3} m and smaller in an element of the active volume of an agitated liquid. Since the densities of the solid and liquid phases are different, particles are not completely entrained by the carrier medium. Therefore, the velocity of a particle in a fixed coordinate system can be written as

$$\bar{w}_p = \bar{w} + \bar{u}. \tag{2}$$

Consequently, to determine the additional flow rate Q_i^{het} it is necessary to determine the relative velocity u of the particles. In general, the displacement of a set of particles of the solid phase depends on the ratio of the densities, the concentration, the shapes and sizes of the particles, and the distribution of the carrier phase currents. The problem of the motion of a solid phase in a moving liquid has so far not been solved [12]. In some instances, however, results adequate for engineering purposes can be obtained by analyzing the motion of the particles relative to the liquid under the assumption that a reference system fixed in the surrounding medium is inertial. The force on an individual particle of density ρ_p and volume V is

$$F = F_l + F_r + m_{ad}\frac{d\bar{u}}{d\tau} - F_{in} + F_{ex} \tag{3}$$

If we assume that the particle is a sphere of radius r_p, and that the segment of its trajectory under consideration is rectilinear, the equation of motion can be written in the form

$$\rho_p V\frac{d\bar{u}}{d\tau} = \rho V\frac{d\bar{w}}{d\tau} + F_r + \rho\frac{V}{2}\frac{d\bar{u}}{d\tau} - 6r_p\sqrt{\pi\mu\rho}\int_0^{\tau}\frac{d\bar{u}}{dt}\frac{dt}{\sqrt{t-\tau}}. \tag{4}$$

Let us consider first laminar flow, for which the resistance of a particle can be determined by Stokes' law. Then, neglecting nonstationary terms, and dividing (4) by $(\rho_p + \rho/2)V$, we obtain the following solution:

$$u \approx u_{st}(1 - e^{-t/\tau}) \tag{5}$$

where

$$\tau = \frac{\left(\rho_p + \frac{\rho}{2}\right)V}{6\pi\mu r_p}. \tag{6}$$

Taking $r_p \sim 10^{-3}$ m, $\rho_p \sim 10^3$ kg/m^3, and the viscosity of the suspension $10^{-2} - 10^{-1}$ Pa·sec (the most common values for carrying out reactions in mixers), we find that the time for the velocity of a particle to reach a steady value u_{st} is 0.5-1 sec. Of course during this time the nonstationary terms in Eq. (4) exert an appreciable effect on the motion.

For turbulent flow of the liquid, and particle sizes smaller than the internal scale of turbulence, the particles execute fluctuating motions in the liquid. The period of velocity fluctuations is approximately equal to that of the liquid [13]. We assume that there is locally isotropic turbulence in the volume element under consideration. Then, according to [14], the internal scale of turbulence is

$$\lambda_0 \sim \frac{(\mu_{\text{ж}}/\rho)^{3/4}}{\varepsilon_\rho^{1/4}}, \tag{7}$$

which for $\varepsilon_\rho \sim$ (1-10) W/kg gives $\lambda_0 \sim (10^{-2}$-$10^{-3})$ m. Hence it follows that the particle sizes we are considering are smaller than the internal scale of turbulence. Therefore, the time for the velocity of a particle to become steady is approximately equal to the period of velocity fluctuations

$$T \sim \frac{1}{\omega_M \mathrm{Re}^{0/4}}. \tag{8}$$

Since the frequency of revolution of mixers for developed turbulent flow $\omega_M \sim 10$/sec, and the Reynolds number Re $\sim 10^4$ and larger, $T \sim 10^{-4}$ sec. Consequently, for turbulent flow the period of random variation of the direction of the fluctuations is $\sim 10^{-4}$ sec, and the large-scale displacement of a particle is essentially determined by the magnitude of the average relative velocity u.

Since we are interested in the magnitude of the displacement between cells, we consider the limits of applicability of the assumption of quasisteady motion. The path length traversed by a particle in time t is

$$l = u_{st}t - u_{st}\tau(1 - e^{-t/\tau}). \tag{8a}$$

Analysis of (8a) shows that for $t > 2\tau$ (or 2T) the large-scale displacement of a particle practically follows the law of steady motion with a velocity u_{st}. Therefore, in investigating the mixing of solid particles, the time step in calculating the model [8] was taken longer than 2 sec, and the radial and axial dimensions of the cells were chosen no smaller than $2u_{st\ max})\tau$. Since the maximum values of $u_{st\ max}$ did not exceed 2×10^{-2} m/sec, the latter requirement is also easily satisfied in devices having a capacity of 0.5 m^3 or more.

Generally speaking, particles move along curvilinear trajectories, but the components of the motion of a particle along any axis obey the same equations as for rectilinear motion. The motions along different axes are independent of one another, at least for sufficiently small Reynolds numbers, which is satisfied in the cases under consideration.

The displacement of a particle relative to a moving liquid can be separated into two components — a vertical component which depends on gravity, and a radial component due to the centrifugal force $-w_\varphi^2/r$ resulting from the curvature of the trajectory of the carrier phase. We write the equations of the average relative motion of a particle in the form

$$F_{r,i} - F_{ex,i} = 0. \tag{9}$$

Using the relation between the Archimedes number and the resistance for various conditions of flow around a spherical particle [14], and taking account of Eq. (9), we obtain the expressions for the velocity components along the z and r axes listed in Table 1. The effect of the concentration (constraint of motion) is taken into account in these expressions by the factor b(c) in accordance with [15]. By substituting these expressions into (1) we can calculate the additional flow rates $Q_{i,k}^{(het)}$, and thus close the mass transfer equations of a cell:

$$\frac{dc_k}{d\tau} = \frac{\sum_{i=1}^{i=3}(Q_{i,k}^{(2)} + Q_{i,k}^{(het)})}{V_k} c_k - \frac{\sum_{i=1}^{i=3}(Q_{i,k}^{(1)} + Q_{i,k}^{(het)}) c_{i,k}^{(1)}}{V_k}. \tag{10}$$

The flow rates of the carrier phase are calculated by using the model in [9].

Experiments on the kinetics of suspension were performed in 0.5 to 3 m diameter vessels with paddle stirrers. Particles from 0.5 to 1 mm in size were mixed at an average concentration of the suspension up to 10 mass % with a density from 865 to 1400 kg/m^3 and a dynamic viscosity from 1.8×10^{-2} to 0.7 Pa·sec. The distribution of the solid phase was determined from its concentration in selected samples.

Analysis of the model of the motion of a heterogeneous medium in mixers [10] showed that under the influence of circulatory flows of the carrier phase the particles are gradually distributed over the volume of the apparatus. The uniformity of the particle distribution in the cells can reach $(0.8\text{-}1.2)c_{i,k}/c_{av}$. After continued operation of the mixer, the particles were still suspended in practically the whole volume of the vessel, but the homogeneity of their distribution decreased as a result of separation, leading to the formation of a ring-shaped

TABLE 1. Relative Velocity of Solid Particles

Conditions of motion	Axial component	Radial component
$Ar \leqslant 36$	$u_z = 0{,}22\dfrac{r_p^2 \Delta\rho g}{\mu_s} b(c)$	$u_r = 0{,}22\dfrac{r_p^2 \Delta\rho r\omega^2}{\mu_s} b(c)$
$36 \leqslant Ar \leqslant 8{,}3\cdot 10^4$	$u_z = 3{,}5\left(\dfrac{\Delta\rho}{\rho_p}\right)\left(\dfrac{r_p^{1,6} g}{\rho^{0,4}\mu_s^{0,6}}\right)^{0,7} b(c)$	$u_r = 3{,}5\left(\dfrac{\Delta\rho}{\rho_p}\right)\left(\dfrac{r_p^{1,6} r\omega^2}{\rho^{0,4}\mu_s^{0,6}}\right)^{0,7} b(c)$
$Ar \geqslant 8{,}3\cdot 10^4$	$u_z = 2{,}45\left(\dfrac{\Delta\rho r_p g}{\rho_p \rho}\right) b(c)$	$u_r = 2{,}45\left(\dfrac{\Delta\rho r_p r\omega^2}{\rho_p \rho}\right)^{0,5} b(c)$

Note. The coefficient of constraint of the motion of a set of particles b(c) is: $b(c) = \begin{cases} 0{,}123\, c^3/(1-c), & c \geqslant 0{,}7, \\ 10^{-1,82(1-c)} c^2, & c \leqslant 0{,}7. \end{cases}$

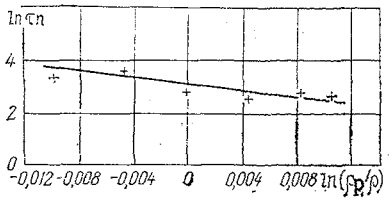

Fig. 1

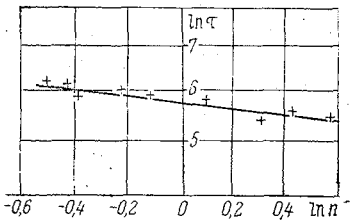

Fig. 2

Fig. 1. Dependence of suspension time on the ratio of densities of solid and liquid phases in a device 1 m in diameter (curve is calculated, points are experimental).

Fig. 2. Effect of number of revolutions of mixer on suspension time in a device 1 m in diameter (curve is calculated, points are experimental).

region in the central part of the apparatus with a depleted solid phase. This phenomenon is particularly noticeable in small devices for large values of $(\rho_p - \rho)/\rho$ and the frequency of revolution of the stirrer. In large devices ($V_a > 3$ m^3) solid particles accumulate along the walls for $\rho_p > \rho$, or near the axis for $\rho_p < \rho$, and in devices up to 1 m in diameter particles accumulate along the walls, in the upper region, and near the mixer shaft. Large vessels and moderate rates of rotation of the stirrers are the most suitable for decreasing the suspension time and increasing the uniformity of the particle distribution.

The effect of the density ratio on the suspension time is shown in Fig. 1, from which it follows that $\tau n \sim (\rho/\rho_p)^{0.7}$, which is confirmed by experiment. To compare our results with those of other researchers, we converted our values to the form $\tau n = f(\Delta\rho/\rho)$, and found that $\tau n \sim (\Delta\rho/\rho)^{0.45}$. This result is close to the value $(\Delta\rho/\rho)^{0.5}$ found in [5], and agrees with data in [3]. It should be noted that in mixing with propeller mixers the exponent of $(\Delta\rho/\rho)$ increases to 0.6 [1] and even to 0.8 [2] as a result of the large axial flow rates in these mixers. The solution of model (10) for mixers with baffle plates showed an increase in the exponent of $(\Delta\rho/\rho)$ to 0.65, which correlates with the increase in axial velocity of an agitated medium when baffle plates are used [10].

The effect of the number of revolutions on the suspension shown in Fig. 2 gives $\tau n \sim n^{0.35}$, which is in satisfactory agreement with experiment. The effect of the basic parameters on the kinetics of suspension can be written in the general form

$$\tau n \sim n^{0,35}\left(\frac{\rho}{\rho_p}\right)^{0,7} Г_D^{1,5}. \qquad (11)$$

The size of the mixing tank, the position of the impeller, and the viscosity of the medium do not have a unique effect, which partly accounts for the considerable difference in the equa-

tions derived in [1-7] for the number of revolutions. Thus, if the volume of the apparatus is increased to 1 m^3 and the impeller is raised to $\Gamma_H = 0.7$,

$$\tau n \sim \Gamma_H^{0.9} V_a^{0.4},$$

whereas a further change of these parameters leads to a sharp change in the exponent of Γ_H; for $\Gamma_H > 1$ and $\rho_p < \rho$, it increases to 3.

Thus, by using the model of macromixing of heterogeneous media [10] and the hydrodynamics of mixers [9], we can investigate the kinetic laws of suspension of small solid particles. Taking account of the possibility of separation, in making an expedient choice of a variant it is necessary to analyze Eqs. (10), varying the size of the apparatus, the position of the impeller, and the conditions of its operation for given physical and chemical properties of the system being modified.

NOTATION

$Ar = 8r_p^3\rho\Delta\rho g/\mu^2$, Archimedes number; $\Gamma_D = D/d_M$ and $\Gamma_H = H/d_M$, simplexes of geometrical similarity; D, diameter of apparatus, m; H, height of filling, m; F, force, N; N, power, W; Q, flow rate, m^3/sec; T, period, sec; V_a, reactor volume, m^3; V_c, cell volume, m^3; V, particle volume, m^3; c, concentration, kg/m^3; d_M, mixer diameter, m; g, acceleration due to gravity, m/sec^2; l, path length, m; m, mass, kg; n, number of rps; r, radius, m; t, time, sec; u, relative velocity of particle, m/sec; w, velocity of liquid, m/sec; ε, power, W; ρ, density kg/m^3; τ, time, sec; λ_0, scale of turbulence, m; μ, dynamic viscosity, Pa·sec; σ, surface, m^2; ω, frequency of revolution, sec^{-1}. Subscripts: i, φ, r, z, directions of corresponding coordinates; k, cell number, v, specific volume; ex, external; l, liquid; in, inertial; M, mixer; ad, additional; s, suspension; r, resistance; av, average; st, steady; p, particle; ρ, specific mass; het, solid phase; (1), at inlet; (2), at outlet.

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